Smooth Planning for Free-floating Space Robots Using Polynomials *

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***Abstract*–Free-floating space manipulator systems, have spacecraft actuators turned off and exhibit nonholonomic behavior due to angular momentum conservation. A path planning methodology for planar free–floating space manipulator systems is developed that allows simultaneous manipulator end-point and spacecraft attitude control using manipulator actuators only. The method is based on mapping the angular momentum to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials are employed driving the system to a desired configuration. It is shown that the method allows for smooth configuration changes in finite and prescribed time, without requiring small cyclical motions. Limitations are discussed and examples are presented.**

***Index Terms*–Space free-floating robots, nonholonomic planning, underactuated systems.**

I. INTRODUCTION

Space robots are already playing important roles in space missions because of their ability to act in environments that are inaccessible or too risky for humans. Free-flying space robots consist of an on-orbit spacecraft fitted with one or more robotic manipulators see Fig. 1. In these, thruster jets can compensate for manipulator induced disturbances, but their extensive use limits a system's useful life span. If the spacecraft thrusters are not operating, as for example during capture operations, these operate in a free–floating mode. Here, dynamic coupling between the manipulator and the spacecraft exists, and manipulator motions induce disturbances to the spacecraft. This mode of operation is feasible when no external forces or torques act on the system and when the total momentum of the system is zero.

A free-floating space robot exhibits a nonholonomic behavior due to the nonintegrability of the angular momentum, [1]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called the Virtual Manipulator, [2]. Inspired by astronaut motions, they proposed a planning technique, which employs small cyclical manipulator joint motions to modify spacecraft attitude. Papadopoulos and Dubowsky studied the *Dynamic Singularities* of free-floating space manipulator systems, which are not found in terrestrial systems and depend on the dynamic properties

Fig. 1. The ETS-7 free-flying space robot. *NASDA.*

of the system, [1, 3]. They also showed that any terrestrial control algorithm could be used to control end-point trajectories, despite spacecraft motions [3]. Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of spacecraft's attitude and its manipulator joints, [4]. To limit the effects of a certain null space, the authors proposed a *bidirectional* approach, in which two desired paths were planned, one starting from the initial configuration and going forward and the other starting from the final configuration and going backwards. The method is not immune to null space problems and yields non-smooth trajectories that require that the joints come to a stop at the switching point.

In another attempt to plan a space robot's motion, Papadopoulos proposed a method that allowed Cartesian motion of the manipulator from an initial point to a final point avoiding dynamic singularities, [5]. The method involved small end-effector Cartesian cyclical motions designed to change the attitude of the spacecraft to one that was known of avoiding dynamic singularities, [5], [6]. Recently, Franch et al. have employed flatness theory to plan trajectories for free-floating systems. Their method requires selection of robot parameters so that the system is made controllable and linearizable by prolongations, [7].

In this paper, a path planning methodology in joint space for planar free-floating space manipulator systems is developed that allows endpoint Cartesian location control, and simultaneous control of the spacecraft's attitude. The method is based on mapping the nonholonomic constraint to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials are employed, driving the system to a desired configuration in finite and prescribed time. Limitations on reaching arbitrary final configurations are discussed and examples are presented.

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II. Dynamics of Free-Floating Space Manipulators

A free-floating space manipulator system consists of a spacecraft (base) and a manipulator mounted on it, as shown in Fig. 2. When the system is operating in free-floating mode, the spacecraft's attitude control system is turned off. In this mode, no external forces and torques act on the system, and hence the spacecraft translates and rotates in response to manipulator movements.

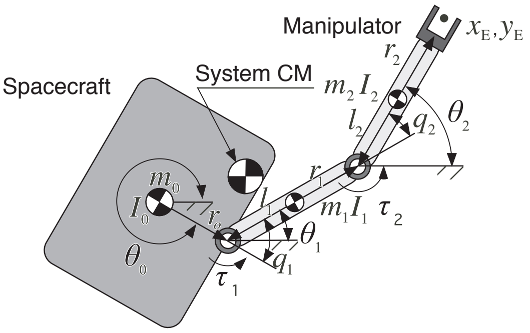

Fig. 2. A Free-floating space manipulator system.

For simplicity, the manipulator is assumed to have revolute joints and an open chain kinematic configuration, so that, in a system with N -degree-of-freedom (dof) manipulator, there will be $N+6$ dof. Since no external forces act on the system, and the initial momentum is zero, the system Center of Mass (CM) remains fixed in space, and the coordinates origin, O, can be chosen to be the system's CM. This frame is considered as inertial since motion durations are much less than the orbital period. The equations of motion for such system have the form, [3],

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is a $N \times N$ positive definite symmetric matrix, called the reduced system inertia matrix, and $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ contains nonlinear velocity terms. The $N \times 1$ column vectors $\mathbf{q}, \dot{\mathbf{q}}$ and $\boldsymbol{\tau}$ represent manipulator joint angles, velocities, and joint torques. In (1), the spacecraft attitude and position do not enter because the system kinetic energy does not depend on base attitude or position nor on its linear or angular velocity when the initial momentum is zero. The base attitude is computed using the conservation of angular momentum, [1],

$${}^{\mathbf{0}}\boldsymbol{\omega}_{\mathbf{0}} = -{}^{\mathbf{0}}\mathbf{D}^{-1}\,{}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}\dot{\mathbf{q}} \tag{2}$$

where ${}^{\mathbf{0}}\boldsymbol{\omega}_{\mathbf{0}}$ is the base angular velocity in the spacecraft 0^{th} frame, and ${}^{\mathbf{0}}\mathbf{D}$, ${}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}$ are inertia-type matrices.

For simplicity, we focus on a free-floating robotic system consisting of a two-dof manipulator mounted on a three-dof spacecraft. The spacecraft is constrained to move in the plane perpendicular to the axes of manipulator joints. For this system, (2) is written as,

$$\begin{gathered} D_0\dot{\theta}_0 + D_1\dot{\theta}_1 + D_2\dot{\theta}_2 = 0 \\ (D_0 + D_1 + D_2)\dot{\theta}_0 + (D_1 + D_2)\dot{q}_1 + D_2\dot{q}_2 = 0 \end{gathered} \tag{3}$$

where θ_0, θ_1, θ_2 are spacecraft attitude and manipulator absolute joint angles, and q_1, q_2, are manipulator joint angles. The D_0, D_1 and D_2 are functions of system inertial parameters and of q_1, and q_2, see Fig. 2. The angular momentum, given by (3), cannot be integrated to analytically yield the spacecraft orientation θ_0 as a function of the system's configuration. However, if the joint angle trajectories are known as a function of time, then (3) can be integrated numerically to yield the trajectory for the base orientation. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system; it is not due to kinematics, as is the case of nonholonomic constraints in mobile manipulators.

III. Nonholonomic Path Planning

In this paper, we focus our attention in finding a path for a free-floating space manipulator system, which connects its initial configuration $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in})$ to a final $(\theta_0^{fin}, \theta_1^{fin}, \theta_2^{fin})$ by actuating manipulator joints only. Note that if this is possible, then one can also design a path to change both the attitude and the manipulator endpoint to desired values, since the endpoint location x_E and y_E is given by, [1],

$$\begin{aligned} x_E &= a\cos\theta_0 + b\cos\theta_1 + c\cos\theta_2 \\ y_E &= a\sin\theta_0 + b\sin\theta_1 + c\sin\theta_2 \end{aligned} \tag{4}$$

where, a, b, c are constant terms, functions of the mass properties of the system, [1]. It is well known that this problem is not trivial, since one must satisfy the nonholonomic constraint and achieve a change in a three-dimensional configuration space with only two controls.

Next, a planning methodology is described that allows for a systematic approach in the planning of systems subject to nonholonomic constraints of the form of (3). Using form in the first of (3), the scleronomic constraint can be written in the Pfaffian form,

$$D_0\left(\theta_0,\theta_1,\theta_2\right)d\theta_0 + D_1\left(\theta_0,\theta_1,\theta_2\right)d\theta_1 + D_2\left(\theta_0,\theta_1,\theta_2\right)d\theta_2 = 0 \tag{5}$$

Note that (5) contains three differentials. Planning can be facilitated if this form is transformed to one in which two differentials appear. This is in principle possible and requires finding appropriate functions u, v, w of $\theta_0, \theta_1, \theta_2$, for which the first equation in (3) can be written as [8-10],

$$du + v\,dw = 0 \tag{6}$$

Assuming that this can be achieved, then planning is done as follows. If we choose functions f and g and set,

$$\begin{aligned} w &= f(t) \\ u &= g(w) \\ v &= -g'(w) = -\frac{du}{dw} \end{aligned} \tag{7}$$

then (6) is satisfied identically. Therefore, the planning problem reduces to choosing functions f and g such that they satisfy the values of *u, v,* and *w* at the initial and final time. Once f and g are found, the trajectories for $\theta_0, \theta_1, \theta_2$ or for θ_0, x_E, y_E are found using the inverse transformation from *u, v, w* to $\theta_0, \theta_1, \theta_2$.

Functions f and g can be polynomials, splines, or any other continuous and smooth functions. For example, one possibility is to choose function f as a fifth order polynomial, so that the system initial and final configuration, velocity and acceleration can be specified, and function g as a third order polynomial, so that initial and final system configurations can be specified.

The method of finding the proper functions *u, v, w* is analytically presented in [9]. Note that although more than one transformations can be found, not all of them are equivalent in terms of complexity.

Next, this methodology is applied to a free-floating space manipulator system in which the manipulator is mounted on the spacecraft's CM. A forward transformation for the case where the manipulator is not mounted at the spacecraft CM is available but is very complex. Mounting the manipulator at the spacecraft CM simplifies the resulting transformation and eliminates Dynamic Singularities from the workspace, [1, 3].

It is easy to show that the angular momentum in this case remains non-integrable and therefore the system still exhibits nonholonomic characteristics. The coefficients of the nonholonomic constraint (5) become,

$$\begin{aligned} D_0(\theta_0,\theta_1,\theta_2) &= \alpha_0 \\ D_1(\theta_0,\theta_1,\theta_2) &= \alpha_1 + \alpha_3 \cdot \cos(\theta_1 - \theta_2) \\ D_2(\theta_0,\theta_1,\theta_2) &= \alpha_2 + \alpha_3 \cdot \cos(\theta_1 - \theta_2) \end{aligned} \tag{8}$$

where the coefficients α_i are given by

$$\begin{aligned} \alpha_0 &= \mathrm{I}_0 \\ \alpha_1 &= \mathrm{I}_1 + \frac{l_1^2 m_0 m_1}{m_0+m_1+m_2} + \frac{m_1 m_2 r_1^2}{m_0+m_1+m_2} + \frac{m_0 m_2 (l_1+r_1)^2}{m_0+m_1+m_2} \\ \alpha_2 &= \mathrm{I}_2 + \frac{m_2(m_0+m_1)l_2^2}{m_0+m_1+m_2} \\ \alpha_3 &= \frac{l_2 m_1 m_2 r_1}{m_0+m_1+m_2} + \frac{l_2 m_0 m_2 (l_1+r_1)}{m_0+m_1+m_2} \end{aligned} \tag{9}$$

and all variables in (9) are defined in Fig. 2.

Following the procedure in [9], the following transformation is chosen among the possible ones:

$$\begin{aligned} u(\theta_0,\theta_1,\theta_2) &= \alpha_0 \cdot \theta_0 + \alpha_2 \cdot \theta_2 - \alpha_3 \cdot \sin(\theta_1 - \theta_2) \\ v(\theta_0,\theta_1,\theta_2) &= \alpha_1 + 2\cdot \alpha_3 \cdot \cos(\theta_1 - \theta_2) \\ w\left(\theta_0,\theta_1,\theta_2\right) &= \theta_1 \end{aligned} \tag{10}$$

The inverse transformation is found via straightforward algebraic manipulations and is given by,

$$\begin{aligned} \theta_0 &= \frac{1}{\alpha_0}\left[u - \alpha_2 w - \alpha_2 \cos^{-1}\left(\frac{v-\alpha_1}{2\alpha_3}\right) - \alpha_3\sqrt{1-\left(\frac{v-\alpha_1}{2\alpha_3}\right)^2}\right] \\ \theta_1 &= w \\ \theta_2 &= w + \cos^{-1}\left(\frac{v-\alpha_1}{2\alpha_3}\right) \end{aligned} \tag{11}$$

It is easy to see that the forward transformation given by (10) is defined for any configuration $\theta_0, \theta_1, \theta_2$. On the other hand, the inverse transform given by (11) is defined if and only if the following inequality is satisfied

$$-1 \le \frac{v(\theta_0,\theta_1,\theta_2) - \alpha_1}{2\alpha_3} \le 1 \tag{12}$$

To satisfy the constraints described by (12), additional freedom must be introduced in the planning scheme. A simple way to achieve this is to introduce additional coefficients in the polynomial $u(w)$. These additional coefficients should not affect the satisfaction of the initial and final conditions but should allow one to shape the path in the $u - v - w$ space so as to satisfy (12).

To this end, the polynomial $u(w)$ should be of order higher than four. However, in such a case, the order of the function v increases and makes an analytical approach difficult. Therefore, we allow more freedom by assuming that the final spacecraft orientation θ_0^{fin} is free and we study which orientations are possible from a given initial configuration. With these remarks, we let the function $u(w)$ have the form

$$u(w) = b_4 w^4 + b_3 w^3 + b_2 w^2 + b_1 w + b_0 \tag{13}$$

Because of the third equation (7), $v(w)$ is given by

$$v(w) = -4 b_4 w^3 - 3 b_3 w^2 - 2 b_2 w - b_1 \tag{14}$$

Using the initial and final system configuration and the transformation given by (10), the initial and final conditions for u, v, w are found and the following linear system is obtained with respect to the unknown coefficients b_i, $i = 0,\ldots,3$:

$$\begin{aligned} \sum_{i=0}^{3} b_i w_{in}^i &= u_{in} - b_4 w_{in}^4 \\ \sum_{i=0}^{3} b_i w_{fin}^i &= u_{fin} - b_4 w_{fin}^4 \\ \sum_{i=0}^{3} i b_i w_{in}^{i-1} &= -v_{in} - 4 b_4 w_{in}^3 \\ \sum_{i=0}^{3} i b_i w_{fin}^{i-1} &= -v_{fin} - 4 b_4 w_{fin}^3 \end{aligned} \tag{15}$$

The above system can be solved to yield the b_i, $i = 0,\ldots,3$, as linear functions of b_4 and θ_0^{fin}.

Replacing these coefficients in (14), the polynomial v is written as a function of b_4 and the unknown spacecraft final orientation θ_0^{fin}. The problem reduces to finding a range of values b_4 and a range of orientations θ_0^{fin} which lead to paths that satisfy (12) for all $w \in [w_{in}, w_{fin}]$. These ranges can be found, studying the following function

$$h(w, b_4, \theta_0^{fin}) = \frac{v(w, b_4, \theta_0^{fin}) - \alpha_1}{2\alpha_3} \quad (16)$$

This function is a third-order polynomial with respect to w. Due to the second equation in (10), once the initial and final θ_1 and θ_2 are given, the initial and final values of h are set and known. Also, these two values always are in the range of $[-1,1]$ for $w \in [w_{in}, w_{fin}]$ because (10) is always defined. Note that the values of h along the path are still given by (16). To satisfy (12), $h(w, b_4, \theta_0^{fin})$ must be in the range $[-1,1]$. This function either has no extremes for $w \in [w_{in}, w_{fin}]$, and therefore (12) is always satisfied, or has extremes whose values must be in the range $[-1,1]$. Obviously, some limitations in the reachable configurations may result due to this reason. The application of the planning methodology is illustrated next.

IV. Application Examples

To illustrate the methodology described above, the free-floating space manipulator shown in Fig. 2 is employed. The system parameters used are shown in Table 1.

TABLE 1. SYSTEM PARAMETERS

Body	l_i $[m]$	r_i $[m]$	m_i $[kg]$	I_i $[kgm^2]$
0	1.0	0.0	400.0	66.67
1	0.5	0.5	40.0	3.33
2	0.5	0.5	30.0	2.50

For this system, a, b, and c, in (4), are given by:

$$a = r_0 m_0 (m_0 + m_1 + m_2)^{-1} = 0$$
$$b = (r_1(m_0 + m_1) + l_1 m_0)(m_0 + m_1 + m_2)^{-1} = 0.89m$$
$$c = (l_2(m_0 + m_1))(m_0 + m_1 + m_2)^{-1} + r_2 = 0.97m$$

For the examples presented here, the duration of motion is chosen equal to 10 *s*. Increasing or decreasing this time has no effect on the path taken, but increases or decreases the torque requirements and the magnitude of velocities or accelerations. For this system, the reachable workspace is computed to be a hollow disk with an external radius equal to R_{max} = 1.86 *m* and an internal one R_{min} = 0.08 *m*. All workspace points on this hollow disk are, in principle, accessible by the end-effector.

The methodology presented earlier can be used to tackle many important points that either were not possible before, or required a great number of small cyclical manipulator joint or endpoint motions.

A. In the first example, the free-floater has to move its manipulator endpoint to a new location and at the same time change its spacecraft attitude to a desired one. Only manipulator actuators are to be used. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (0^o, 1.5m, -0.5m)$ and the final $(\theta_0, x_E, y_E)^{fin} = (-60^o, 0.0m, 1.0m)$. Next, b_4 is chosen so that first all constraints are satisfied. This requirement may result in a range of possible b_4. Of these, b_4 is chosen so that the range of allowable final spacecraft attitudes is maximized. For example, choosing $b_4 = 1.8$ results in final attitudes $\theta_0^{fin} \in (-65.7^o, -53.4^o)$, which contains the desired -60^o. Fig. 3 depicts snapshots of the free-floater motion as it changes its configuration. The manipulator roughly rotates counter-clockwise, adjusting its inertia appropriately, while the spacecraft rotates in the opposite direction, reaching the desired final attitude.

As shown in Fig. 4, the desired configuration is reached in the specified time. Also, all trajectories are smooth throughout the motion, and the system starts and stops smoothly at zero velocities, as expected. This is an important characteristic of the method employed and is due to the use of smooth functions, such as polynomials.

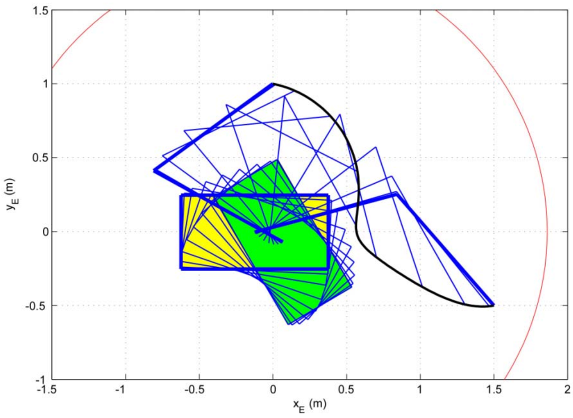

Fig. 3. Snapshots of a free-floater moving to a desired θ_0, x_E, y_E.

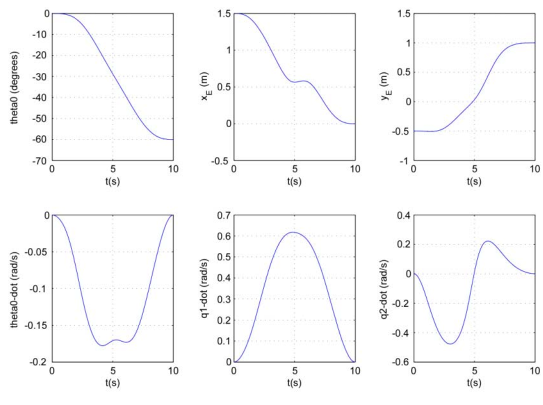

Fig. 4. Configuration variables and spacecraft orientation and joint angles rate trajectories that correspond to snapshots in Fig. 3.

The corresponding joint torques are given in Fig. 5. These torques are computed using (1) and the elements of the reduced inertia matrix, given in [1]. As shown in Fig. 5, the required torques are small and smooth while they can be made arbitrarily small, if the duration of the maneuver is increased. The implication of this fact is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible.

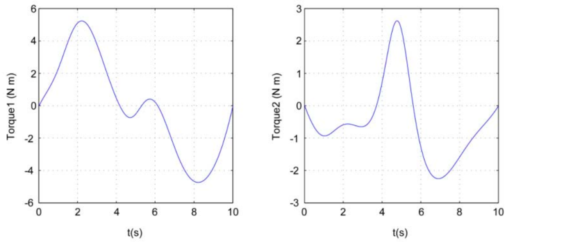

Fig. 5. Manipulator torques required for the motion shown in Fig. 3.

B. In the second example, the task is to change the spacecraft attitude with an appropriate motion of the manipulator. In addition, we require that the manipulator endpoint returns to its initial Cartesian location after the base attitude has been corrected to the desired one. The conditions are as follows: the orientation of the base should change counter clockwise from $\theta_0^{in} = -80^o$ to $\theta_0^{fin} = 0^o$, while the endpoint location is $(x_E,\ y_E)^{fin} = (x_E, y_E)^{in} = (1.79, 0.48)m$. In this case, no b_4 exists that can accomplish this task. However, a solution exists if the spacecraft is allowed to rotate clockwise with $\theta_0^{fin} = -360^o$. Choosing $b_4 = 0.01$, the range of reachable attitudes is $\theta_0^{fin} \in (-474.7^o, -287.7^o)$, which includes $\theta_0^{fin} = -360^o$. The resulting path for this motion is depicted in Fig. 6, while the trajectories of the configuration variables and the rates of the spacecraft orientation and the joint angles, are shown in Fig. 7.

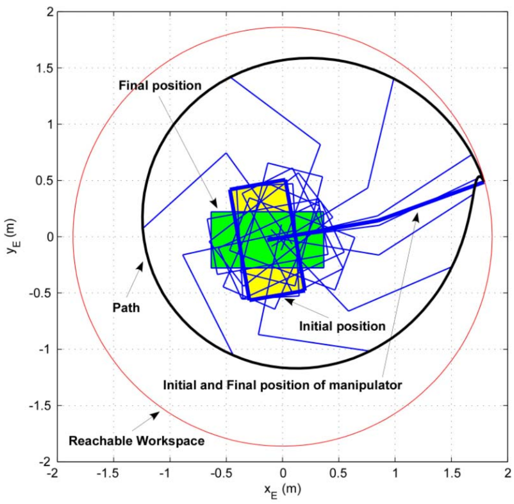

Fig. 6. Snapshots of a free-floater changing its spacecraft attitude.

Again, all trajectories and torques shown in Fig. 8 are smooth. As shown in Fig. 6, the manipulator executes a full turn in Cartesian space and returns to its initial location. During this rotation, the manipulator effective inertia is adjusted such that the spacecraft attitude at the end of the motion is the desired one.

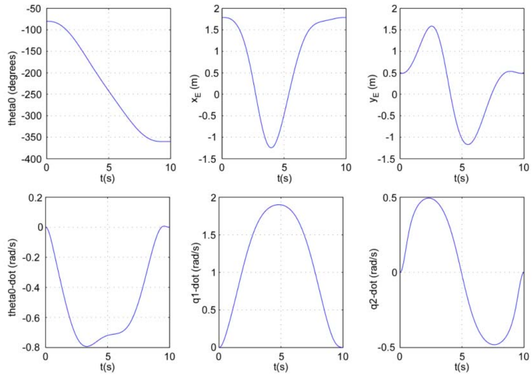

Fig. 7. Configuration variables and spacecraft orientation and joint angles rate trajectories that correspond to snapshots in Fig. 6.

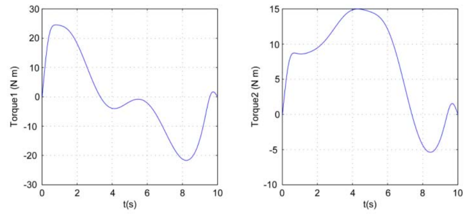

Fig. 8. Manipulator torques required for the motion shown in Fig. 6.

C. In this example, the manipulator end-point is desired to move to a new location, while at the end of the motion the base must be at its initial attitude, i.e. $\theta_0^{fin} = \theta_0^{in}$. Fig. 9 shows snapshots of the free-floater motion when it moves from $(\theta_0, x_E, y_E)^{in} = (0^o, 1.86m, 0m)$ to $(\theta_0, x_E, y_E)^{fin} = (0^o, 0.7m, 0.4m)$. Choosing $b_4 = 70$, the allowed final orientation is in the range $\theta_0^{fin} \in (-5^o, 3.5^o)$. Smooth trajectories for output variables are shown in Fig. 10. Note that as expected, not all configurations are reachable from an initial one. For example, it is not rational to expect large base rotations with a small point-to-point manipulator motion. Similarly, it may not be possible to move to any Cartesian point and have the final spacecraft attitude unchanged. To illustrate this, consider the case, where the system has to move from $(\theta_0, x_E, y_E)^{in} = (0^o, 1.26m, 1.29m)$ to $(\theta_0, x_E, y_E)^{fin} = (0^o, 1.25m, 0.28m)$. In this case, the only possible attitude is achieved for $b_4 = 300$ and is $\theta_0^{fin} = 2.73^o$, i.e. no path can be found. Snapshots of the resulting motion are shown in Fig. 11, while trajectories are shown in Fig. 12.

An interesting issue is the behavior of the method in the presence of joint limits. In principle, joint limits can be treated as obstacles in the joint space and mapped in the $u-v-w$ space as in [9]. The task is then to plan trajectories in the $u-v-w$ so as to avoid these obstacles. This can be achieved in general by increasing the order of the polynomials used. However, this is an issue of current and future research.

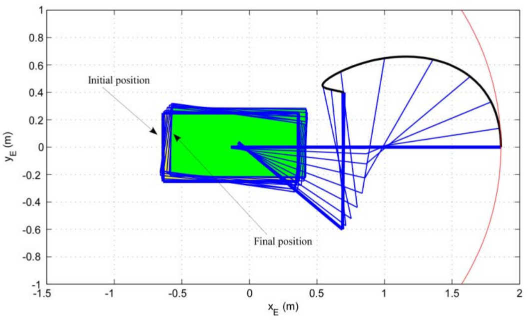

Fig. 9. Snapshots of a free-floater moving to a desired x_E, y_E .

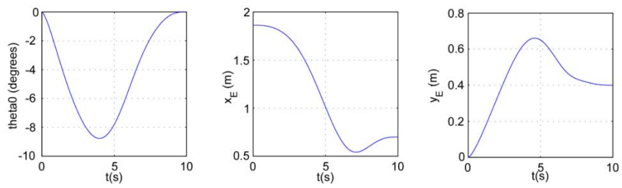

Fig. 10. Configuration variable trajectories that correspond to Fig. 9.

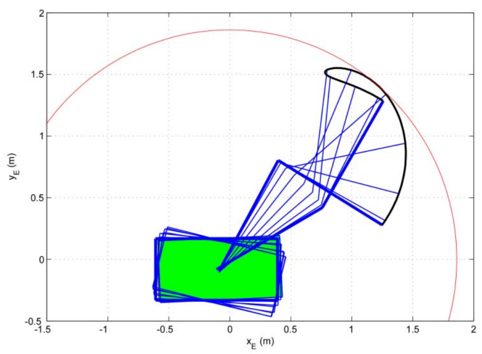

Fig. 11. Snapshots of an unsuccessful attitude change.

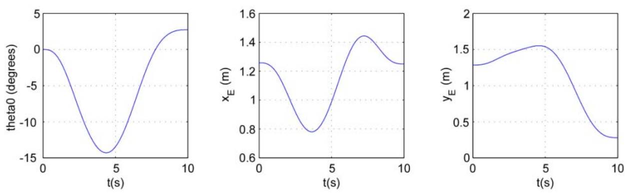

Fig. 12. Configuration trajectories that correspond to Fig. 11.

One should stress out that the results presented here are very interesting from the scientific point of view, because they prove that controlling both the spacecraft attitude and the position of the end-effector does not necessarily require a large number of small cyclical motions, but it can be achieved by pre-planned large motions of the manipulator. This is much more effective than the previous solution to the problem. From the practical point of view, one should investigate the applicability of the method to more than two dof and three-dimensional systems. It is expected that the problem will be more complex in these cases. However, one may always choose three variables, one not actuated and two that are actuated, and apply the techniques presented here. Choosing a sequence of triplet variables can result in more complex attitude and manipulator maneuvers than is currently possible.

As a final comment, recent work shows that configuration accessibility can be improved drastically using a different definition for v . This leads to a wider range for $q_2(t)$ shapes, yielding results even in unsuccessful cases such as the one in Fig. 11.

V. Conclusions

A path planning methodology for planar free–floating space manipulator systems was developed that allows endpoint Cartesian location control, and simultaneous control of the spacecraft's attitude. The method is based on mapping the nonholonomic constraint to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials are employed, driving the system to a desired configuration in finite and prescribed time. The methodology avoids previous solutions that required a large number of small joint space or Cartesian space motions, and therefore were not practical. Limitations on reaching arbitrary final configurations were discussed and illustrative examples were presented.

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